

$$y = a \sin[b(x - c)] + d$$

↓  
 amplitude  
 (height of bump)

↓  
 frequency  
 (helps you get period)

↓  
 horz. shift  
 (beginning endpoint)

↓  
 vert. shift (this is your midline)

$$y = a \cos[b(x - c)] + d$$

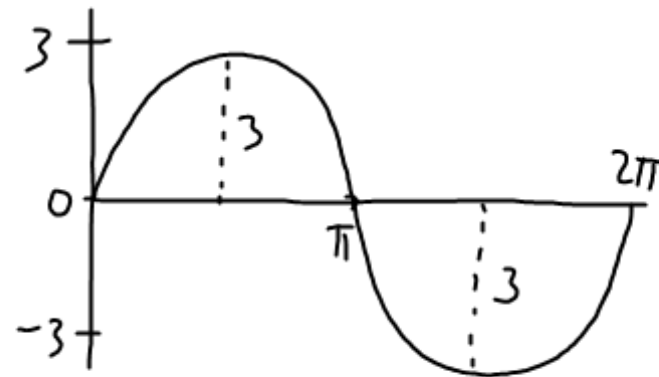
Period - time it takes for 1 complete cycle

$$\text{Period} = \frac{2\pi}{b}$$

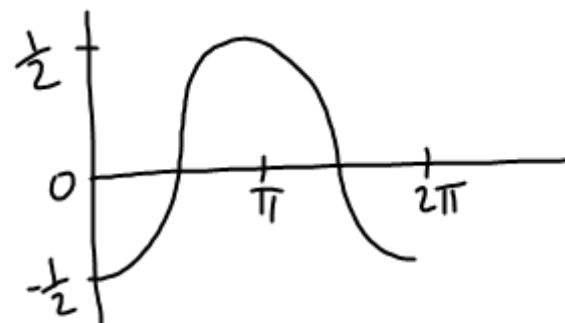
# Vertical Stretch (a)

$y = a \sin x$  the  $a$  is the amplitude or vertical stretch

$$y = 3 \sin(x)$$



$$y = -\frac{1}{2} \cos(x)$$



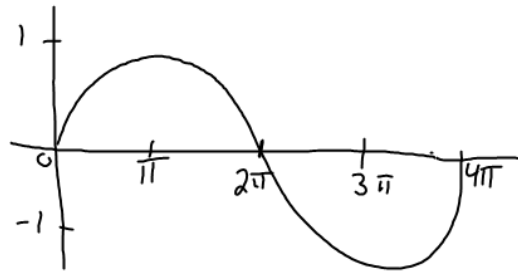
the  $\frac{1}{2}$  changes the amplitude, the negative flips the graph over x-axis

# horizontal stretch/compression (b)

$$y = \sin bx$$

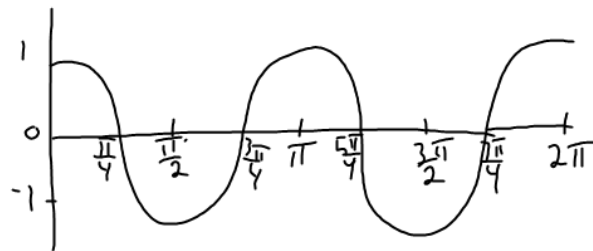
frequency =  $\frac{1}{2}$   
 $y = \sin \frac{1}{2}x \rightarrow$  will do  $\frac{1}{2}$  cycle in  $2\pi$

$$\text{Period} = \frac{2\pi}{b} \Rightarrow \frac{2\pi}{\frac{1}{2}} \Rightarrow 2\pi \cdot \frac{2}{1} = 4\pi$$



$y = \cos 2x$  frequency is 2 so it will do 2 cycles in  $2\pi$

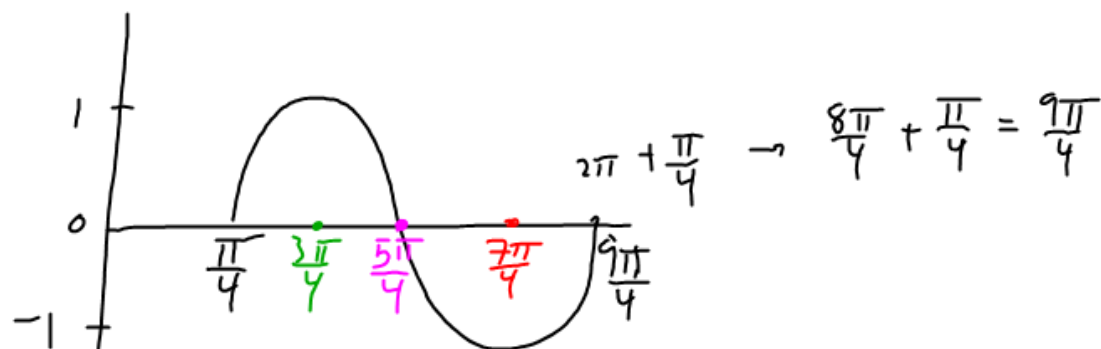
$$\text{Period} = \frac{2\pi}{2} = \pi$$



# horizontal shift (c)

$$y = \sin(x - c)$$

$y = \sin(x - \frac{\pi}{4})$  shifts right  $\frac{\pi}{4}$ , gives you the starting endpoint



$$2\pi + \frac{\pi}{4} \rightarrow \frac{8\pi}{4} + \frac{\pi}{4} = \frac{9\pi}{4}$$

$$\frac{\pi}{4} + \frac{9\pi}{4} = \frac{10\pi}{4} \cdot \frac{1}{2} = \frac{5\pi}{2}$$

$$\frac{\pi}{4} + \frac{5\pi}{2} = \frac{6\pi}{4} \cdot \frac{1}{2} = \frac{3\pi}{2}$$

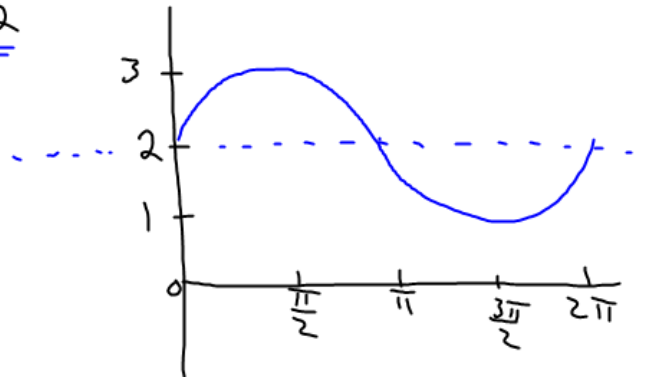
$$\frac{5\pi}{2} + \frac{9\pi}{4} = \frac{14\pi}{4} \cdot \frac{1}{2} = \frac{7\pi}{2}$$

# Vertical shift (d)

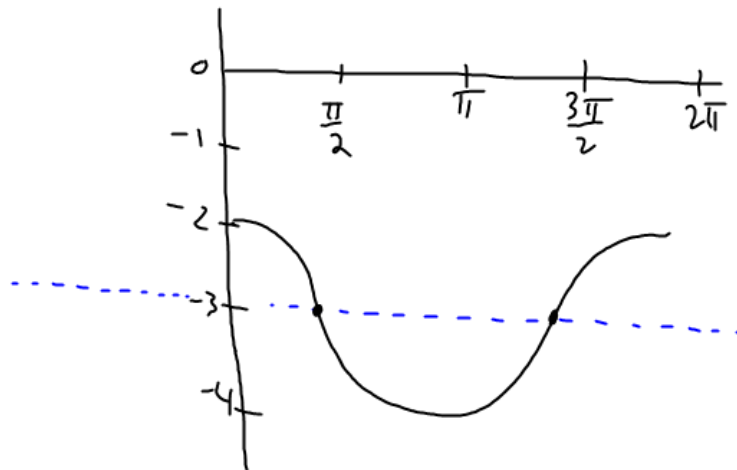
$$y = \sin(x) + d$$

The  $d$ -value gives you the mid-line

$$y = \sin(x) + \underline{\underline{2}}$$

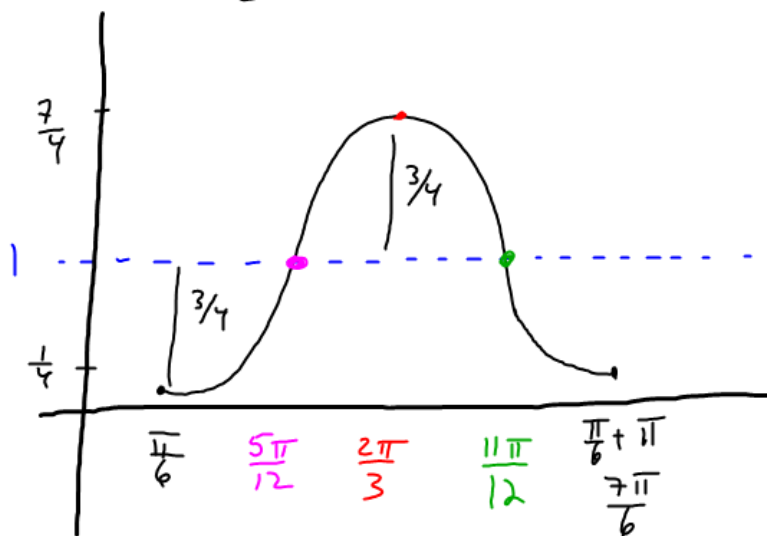


$$y = -3 + \cos(x) \rightarrow y = \cos(x) - 3$$



# Example

$$y = -\frac{3}{4} \cos \left[ 2 \left( x - \frac{\pi}{6} \right) \right] + 1$$



$$P = \frac{2\pi}{6} \Rightarrow \frac{2\pi}{2} = \pi$$

$$\frac{\pi}{6} + \frac{2\pi}{6} = \frac{8\pi}{6} \cdot \frac{1}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\frac{\pi}{6} + \frac{2\pi}{3} = \frac{\pi}{6} + \frac{4\pi}{6} = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$$

$$\frac{2\pi}{3} + \frac{7\pi}{6} = \frac{4\pi}{6} + \frac{7\pi}{6} = \frac{11\pi}{6} \cdot \frac{1}{2} = \frac{11\pi}{12}$$

## My Steps

- ① Draw curve first
- ② Put in midline
- ③ Find Period
- ④ Find the Horiz. Shift to get the starting endpoint
- ⑤ Find the ending endpoint using the start point plus the period
- ⑥ Find x-round. by avg.
- ⑦ Put in axes and find y-round. using the a-term

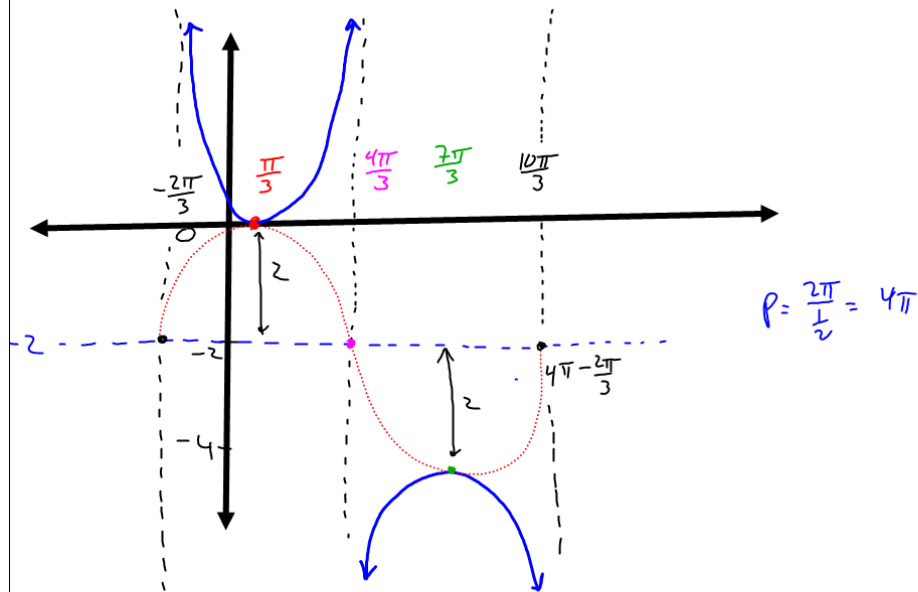


# Secant + cosecant

- Secant matches up with cosine
- cosecant matches up with sine

$\Rightarrow$  For  $\sec x + \csc x$  graph the reciprocal function and use as a guide for the  $\sec x$  and  $\csc x$ . Put in vertical asymptotes wherever the guide crosses midline.

$$y = 2 \csc \left[ \frac{1}{2} \left( x + \frac{2\pi}{3} \right) \right] - 2 \quad \text{graph } y = 2 \sin \left[ \frac{1}{2} \left( x + \frac{2\pi}{3} \right) \right] - 2$$



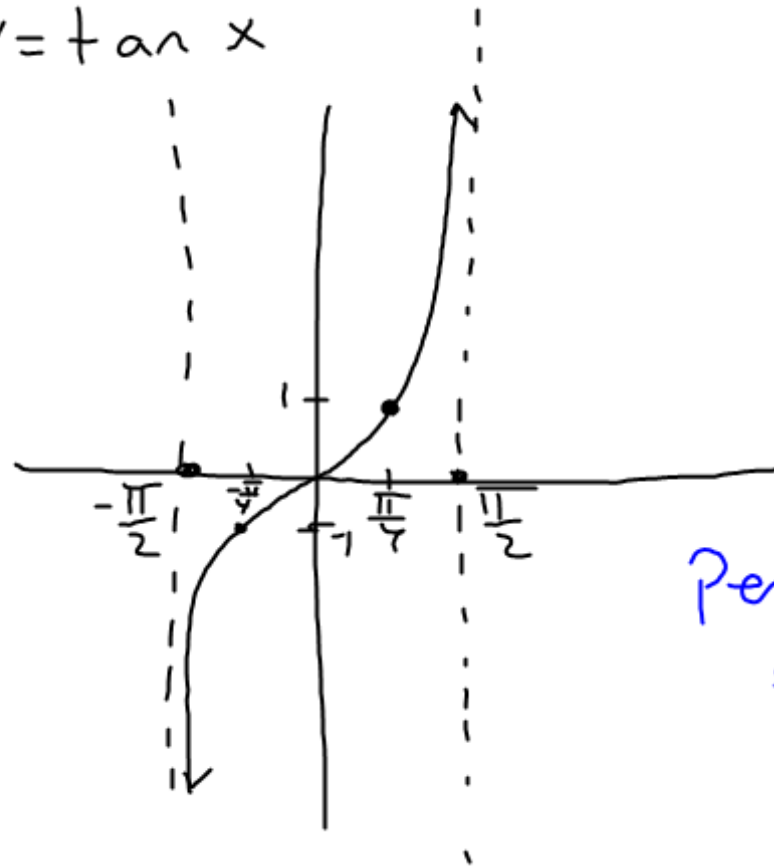
$$-\frac{2\pi}{3} + \frac{10\pi}{3} = \frac{8\pi}{3} \cdot \frac{1}{2} = \frac{4\pi}{3}$$

$$-\frac{2\pi}{3} + \frac{4\pi}{3} = \frac{2\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{3}$$

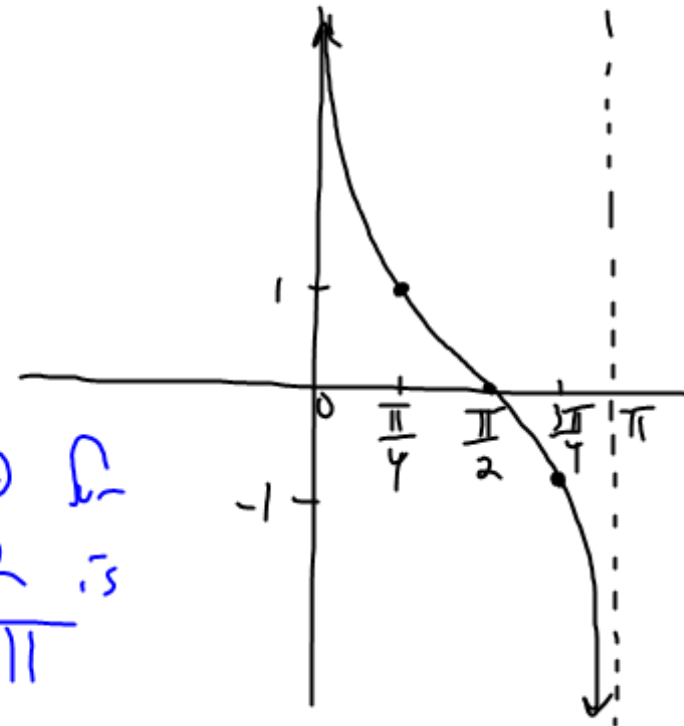
$$\frac{4\pi}{3} + \frac{10\pi}{3} = \frac{14\pi}{3} \cdot \frac{1}{2} = \frac{7\pi}{3}$$

# tangent & cotangent

$$y = \tan x$$



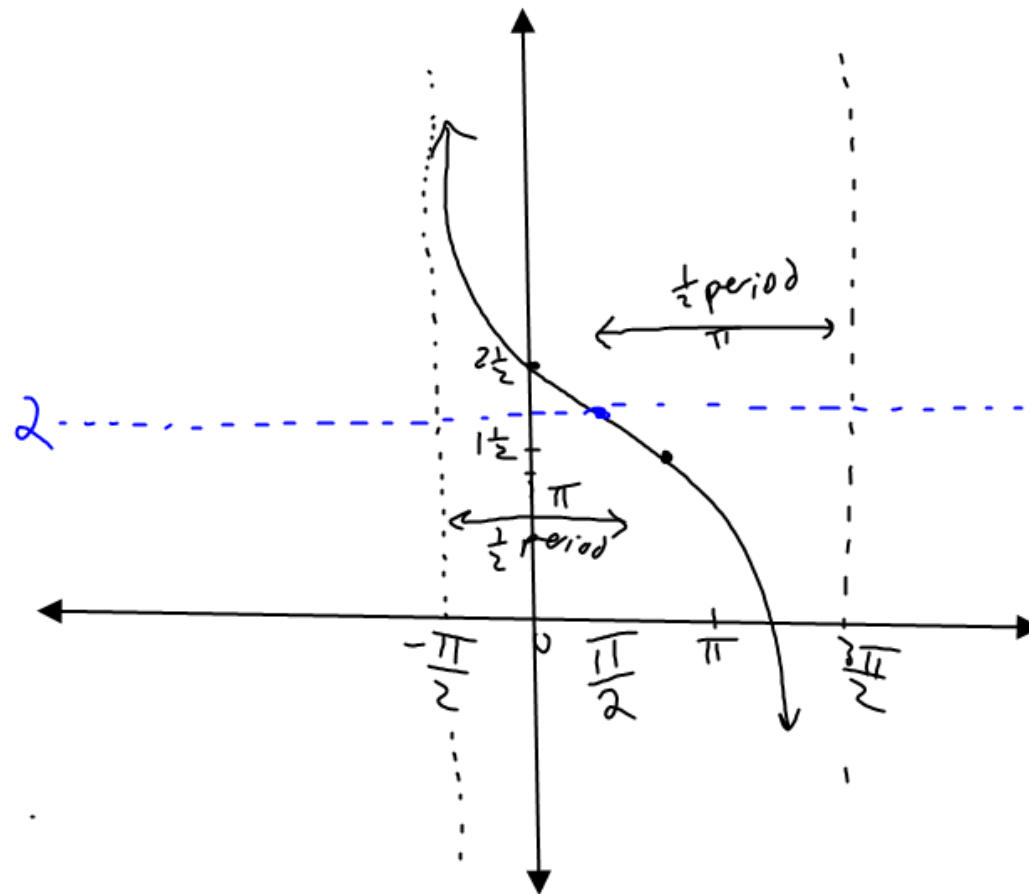
$$y = \cot x$$



Period for  
both is  
 $\pi$

# Example

$$y = -\frac{1}{2} + \tan\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right] + 2$$



① sketch curve

② Find period

$$P = \frac{\pi}{b}$$

③ Find the shift using the center

④ Start point is the center -  $\frac{1}{2}P$

⑤ End point is the center +  $\frac{1}{2}P$

⑥ Put in axes

⑦  $\frac{1}{2}$ -way between center and asymptotes  
Find y-coord. by doing midline plus and minus a-term

The first step to modeling a periodic function with a sinusoid is to identify the highest and lowest points and the period. You can then use that information to write a function  $y = \pm a \cos[b(x-c)] + d$  or  $y = \pm a \sin[b(x-c)] + d$ .

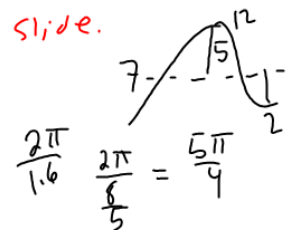
### Example:

A weight on the end of a spring is pulled down to just 2 inches above the floor and then released. After 0.8 sec. it reaches its highest point 12 inches off the floor. Write a function for the height of the weight from the floor with respect to time.

### Analysis:

- The period is the time for 1 complete cycle. In this case it takes 0.8 seconds for  $\frac{1}{2}$  a cycle so it will take 1.6 seconds for a complete period.
- The high point is given to us (0.8, 12).
- The low point is the initial condition in this example (0, 2).
- The equation would be  $y = 5 \cos\left[\frac{5\pi}{4}(x - 0.8)\right] + 7$  using the method on the next slide.

Note:  $\frac{2\pi}{1.6} = \frac{2\pi}{\frac{8}{5}} = \frac{10\pi}{8} = \frac{5\pi}{4}$



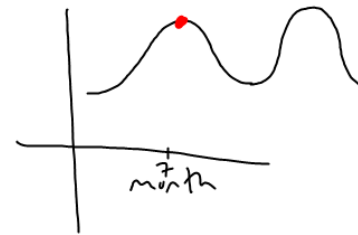
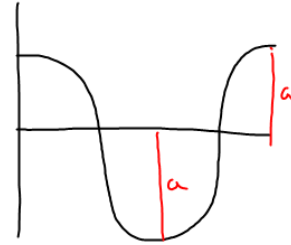
$$y = a \cos[b(x-c)] + d$$

$$a = \frac{\text{high point (y-value)} - \text{low point (y-value)}}{2}$$

$$b = \frac{2\pi}{\text{Period}}, \text{ Period from context}$$

$$c = \text{X-coord. of the high point}$$

$$d = \frac{\text{high point (y-coord)} + \text{low point (y-coord)}}{2}$$



PLEASE CHECK  
YOUR EQUATION

- Plug it in the y=
- look at the table for known points
- or create a scatter plot with given data and graph your function in the same window to see fit.